

# Value Function Learning for AutoRally Racing

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## Summary

- We reframe model predictive control (MPC) as a reinforcement learning (RL) approach where each decision is a control sequence.
- This formalizes value learning approaches which rely on MPC to generate the value function targets.
- We show incorporating a value function can improve racing performance of simulated AutoRally car

## Model Predictive Control as Reinforcement Learning

Given MDP  $\mathcal{M}=(\mathcal{X}, \mathcal{U}, p, c, \gamma)$

- $\mathcal{X}$ : state space
- $\mathcal{U}$ : control space
- $p(x_{t+1}|x_t, u_t)$ : transition probability
- $c(x_t, u_t, x_{t+1})$ : cost function
- $\gamma$ : discount factor

RL problem: Find policy  $\pi(u_t|x_t)$  that minimizes sum of accumulated costs while having high entropy (i.e., noisiness).

$$\min_{\pi} \mathbb{E}_{\pi, p} \sum_{t=0}^{\infty} \gamma^t (c(x_t, u_t, x_{t+1}) - \lambda \mathcal{H}(\pi(u_t|x_t)))$$

Solving this problem may be too difficult, so we form a relaxed RL problem that is potentially easier.

We instead work with a policy  $\tilde{\pi}(u_t, \dots, u_{t+H-1}|x_t)$  that gives feedback every  $H$  steps. The corresponding MDP is  $\tilde{\mathcal{M}}=(\tilde{\mathcal{X}}, \tilde{\mathcal{U}}, \tilde{p}, \tilde{c}, \tilde{\gamma})$ .

- $\tilde{\mathcal{X}} = \mathcal{X}, \quad \tilde{\mathcal{U}} = \mathcal{U}^H, \quad \tilde{\gamma} = \gamma^H, \quad \tilde{\lambda} = \gamma^{H-1}\lambda$
- $\tilde{p}(\tilde{x}_{k+1}|\tilde{u}_k, \tilde{x}_k) = \int \prod_{h=0}^{H-1} p(x_{kH+h+1}|x_{kH+h}, u_{kH+h}) dx_{kH+(1:H-1)}$
- $\tilde{c}(\tilde{x}_k, \tilde{u}_k, \tilde{x}_{k+1}) = \mathbb{E}_{x_{kH+(1:H-1)}|x_{kH}, u_{kH}, x_{kH+H}} \sum_{h=0}^{H-1} \gamma^h c_{kH+h}$

The following RL problem is an upper bound of the original one:

$$\min_{\tilde{\pi}} \mathbb{E}_{\tilde{\pi}, \tilde{p}} \sum_{k=0}^{\infty} \tilde{\gamma}^k (\tilde{c}(\tilde{x}_k, \tilde{u}_k, \tilde{x}_{k+1}) - \tilde{\lambda} \mathcal{H}(\tilde{\pi}(\tilde{u}_k|\tilde{x}_k)))$$

Optimal value function  $\tilde{V}^*$  satisfies the recurrence

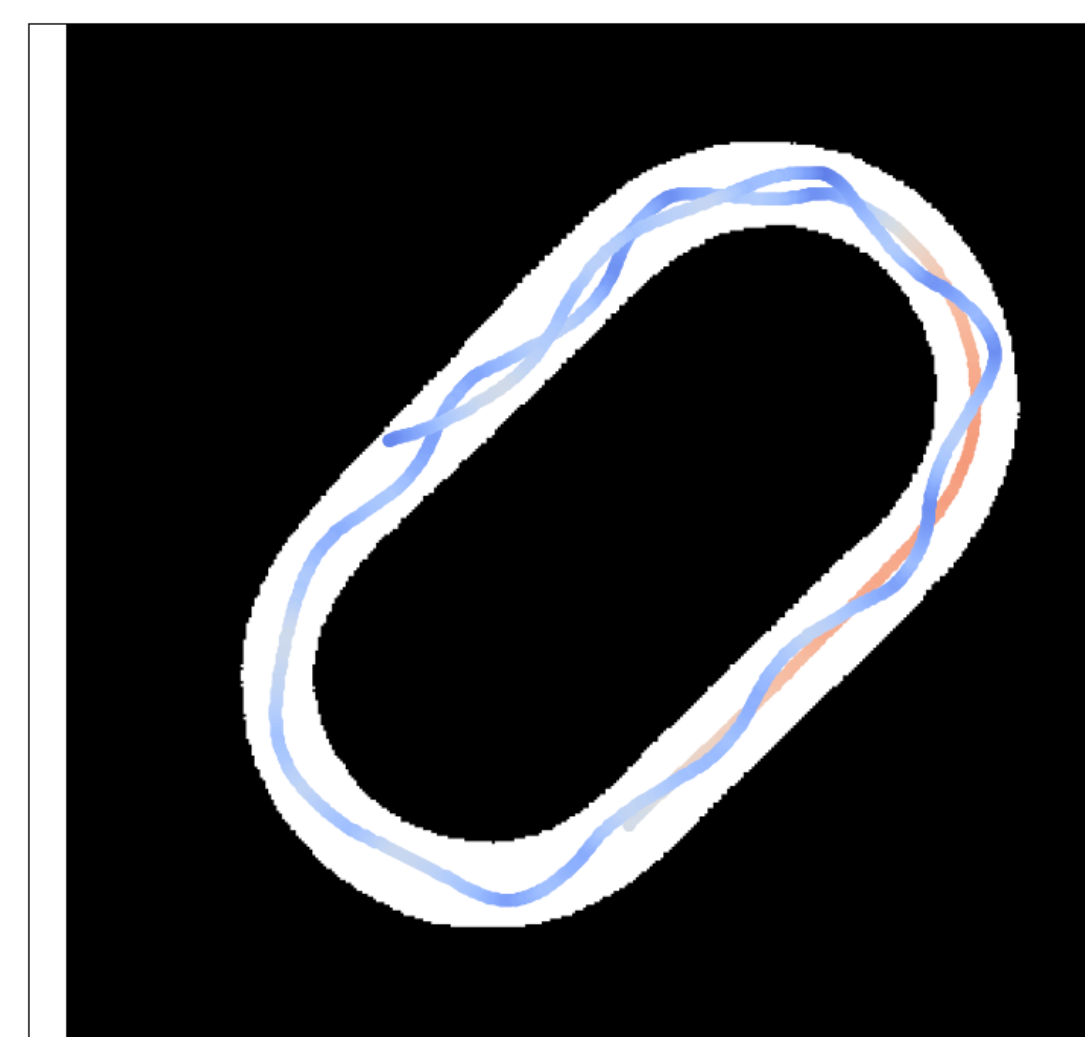
$$\tilde{V}^*(\tilde{x}) = -\tilde{\lambda} \log \int \exp\left(-\frac{1}{\tilde{\lambda}} \mathbb{E}_{\tilde{p}(\tilde{x}'|\tilde{x}, \tilde{u})} [\tilde{c}(\tilde{x}, \tilde{u}, \tilde{x}') + \tilde{\gamma} \tilde{V}^*(\tilde{x}')] \right) d\tilde{u}$$

This recurrence can be used to train an approximator of the optimal value function.

- + Can get accelerated convergence of the value function
- + Value function targets can be efficiently estimated with algorithms like model predictive path integral (MPPI)
- Some performance degradation since we represent policy as  $H$ -step open loop distribution

## Simulated Results

- Cost function encodes that car minimizes lap time while not crashing
- Control algorithm: MPPI used in receding horizon
- We vary planning horizon and whether we use value function as terminal cost
- Value function is a 2-layer neural network trained with an optimization horizon of 0.5 seconds
- Value function can improve performance and even stabilize car in case of short planning horizon



Planning horizon = 0.5 seconds  
No value function used  
Lap time:  $12.56 \pm 1.90$  s  
5 crashes in 5 trials



Planning horizon = 0.5 seconds  
Value function used  
Lap time:  $6.43 \pm 0.29$  s  
2 crashes in 5 trials



Planning horizon = 1 second  
No value function used  
Lap time:  $7.28 \pm 0.42$  s  
0 crashes in 5 trials



Planning horizon = 1 second  
Value function used  
Lap time:  $6.50 \pm 0.22$  s  
0 crashes in 5 trials

